TWO OR THREE?

ROBERT ALEXY, KIEL

I. THE QUESTION

In the last 15 lines of an article on Arthur Kaufmann’s theory of law-making, published in 2005, I argued or, better – the 15 lines comprise neither argument nor explanation – made the conjecture that there exists, alongside subsumption and balancing, a third basic operation in the application of law: analogy between or a comparison of cases.¹ The formal structure of subsumption may be represented in a deductive scheme, the subsumption formula, and the formal structure of balancing may be represented by an arithmetic scheme, the weight formula.² In the Kaufmann article, I suggest that analogy can be seen as a third scheme. I have attempted to capture its fundamentals by means of two diametrically opposed rules, which, slightly modified with respect to their formal representation, run as follows:

A₁: In every case cᵢ, each case cⱼ may be adduced with the argument that cᵢ shares with cⱼ the features F₁, ⋯, Fₙ, and that cᵢ, for that reason and because the rule F₁, ⋯, Fₙ → Q is valid, ought to be treated, as cⱼ, to the effect that Q.

A₂: In every case in which an argument of the form A₁ is put forward, it may be claimed that cᵢ is distinguished by the features F₁, ⋯, Fₙ from cⱼ, and that cᵢ, for that reason and because the rule F₁, ⋯, Fₙ → ¬Q is valid, ought not, in contradistinction to cⱼ, to be treated to the effect that Q.³

This proposal has been criticized by Bartosz Brożek and Carsten Bäcker. Both argue that A₁ and A₂ do not adequately represent analogical reasoning. A main target of their objections turns on the rules F₁, ⋯, Fₙ → Q and F₁, ⋯, Fₙ → ¬Q. Brożek maintains that if F₁, ⋯, Fₙ → Q is a ‘valid legal rule’, then, as he puts it, ‘the rule applies directly and explicitly to both cases’.⁴ This, however, would mean that ‘there is no need for analogical reasoning here’.⁵ Bäcker expresses a similar objection, contending that the point of A₁ is not that the two cases are similar but that there exists a single rule that applies to both of them.⁶ The fact that the two rules in A₁ and A₂ lead to a contradiction is held to be the result of an ordinary conflict between rules,

---

⁵ Ibid.
⁶ Carsten Bäcker, Begründen und Entscheiden (Baden-Baden: Nomos 2008), 298.
which is to be resolved by means of balancing. A comparison of cases plays no essential role here.  

Brożek adds two objections. First, the analogy scheme says nothing about how to resolve the conflict that stems from the application of both $A_1$ and $A_2$, and, second, it also says nothing about how comparison or analogy is related to the other two basic operations, that is, subsumption and balancing.

II. THE IDEA OF A BASIC OPERATION

The question of whether there exist two or three basic operations presupposes that it makes sense to speak about basic operations in the application of law at all. Why, to begin with, should we conceive of subsumption as a basic operation? The answer is that subsumption is an operation that necessarily has to be performed according to some version or other of the general scheme that governs all cases in which legal rules are to be applied. This scheme runs as follows:

\[
\begin{align*}
(1) \ & \forall x \ (Tx \rightarrow ORx) \\
(2) \ & \forall x \ (M^1x \rightarrow Tx) \\
(3) \ & \forall x \ (M^2x \rightarrow M^1x) \\
& \vdots \\
(n + 2) \ & \forall x \ (Sx \rightarrow M^n x) \\
(n + 3) \ & Sa \\
(n + 4) \ & ORa \\
\end{align*}
\]

This scheme, which might be called the ‘subsumption formula’, has three distinctive characteristics that qualify it as a basic scheme. It is formal, necessary, and specific. Its specific character stems from the fact that it unfolds according to a specific kind of rule, in this case the rules of logic. It is, second, necessary, because it must be employed, in one version or another, in all cases in which legal rules are to be applied, and, third, it is completely formal. This last point implies that the subsumption scheme stands in need of saturation by means of substantial arguments that in most cases have a structure distinct from that of subsumption. These further arguments may well comprise balancing and comparison. The fact, however, that subsumption qua basic form of argument is necessarily connected with arguments of other forms does not in any way deprive the scheme of its basic character. The

---

7 Ibid., 299.
8 Brożek, ‘Analogy in Legal Discourse’ (n. 4), 199.
9 Alexy, ‘On Balancing and Subsumption’ (n. 2), 434.
12 Alexy, ‘Arthur Kaufmanns Theorie der Rechtsgewinnung’ (n. 1), 64.
idea of a basic operation or a basic scheme is compatible with even the most radical versions of holism. All of this bears, too, on balancing. The basic scheme of balancing is the weight formula:

\[ W_{i,j} = \frac{I_i \cdot W_i \cdot R_i}{I_j \cdot W_j \cdot R_j} \]

This formula represents the core of a complex argument-structure. In standard cases, where only two principles are involved, balancing begins with the subsumption of the case under two competing principles \((P_i, P_j)\), and continues with an assignment of values, first, to the intensity of interferences \((I_i, I_j)\) with \(P_i\) and \(P_j\), second, to the abstract weights \((W_i, W_j)\) of both principles, and, third, to the degree of reliability of the empirical assumption \((R_i, R_j)\) respecting what the measure in question means for the non-realization of \(P_i\) and the realization of \(P_j\). Once numbers are assigned to these variables, calculation of the concrete weight of \(P_i\) \((W_{i,j})\) is no more difficult than deduction, once the class of premises is complete. Let us assume that under the circumstances of the case \((C)\), the concrete weight \((W_{i,j})\) of \(P_i\) is greater than \(1\). Then, takes precedence over \(P_j\) under the circumstances of the case \((C): (P_i \quad P_j) \quad C\). According to the law of competing principles, this means that a rule is valid that has \(C\) as its protasis and the legal consequences \((Q)\) of \(P_i\) in the concrete case as its apodosis: \(C \rightarrow Q\). The case can now be subsumed under this rule. This shows that subsumption stands not only at the beginning of balancing but also at the end.

Again, this connection of balancing with subsumption by no means deprives balancing of its basic character. Balancing works, first, according to a specific kind of rule, in this case the rules of arithmetic, second, it must be employed in all cases in which legal principles are to be applied, that is, it is necessary, and, third, it is formal, because it can be connected, in principle, with all arguments of all other forms. The question is whether this also applies to analogy or comparison. Does the comparison of cases also have a formal, necessary, and specific character in the way subsumption and balancing do? This depends on the structure of the comparison of cases.

### III. The Structure of the Comparison of Cases

Brożek uses variations of H.L.A. Hart’s famous vehicles-in-the-park case as examples. I shall do so as well, albeit with some further modifications. We begin with the assumption that the local authorities have issued the rule in Hart’s example:

---


14 On the details see Alexy, ‘On Balancing and Subsumption’ (n. 2), 444–5.


(R₁) ‘[N]o vehicle may be taken into the park.’

In case of an unruly automobile driver, a court will have already applied the rule. The following rule may therefore be considered as established by both legislation and precedent:

(R₂) No automobile may enter the park.

The same court, however, has declared that bicycles have to be rendered exempt from the rule (R₁), for cycling, first, is a recreational activity protected by freedom of action and, second, it causes neither pollution nor noise. A certain level of danger for pedestrians is recognized, but it is deemed to be rather low. The following rule may therefore be considered as an established rule stemming from the case law:

(R₃) Bicycles may enter the park.

Now, a new case appears before our court. A rider of a motor scooter is accused of having violated the vehicle rule (R₁). The rider argues that riding a motor scooter in the park is so similar to riding a bicycle that it, too, must be allowed. Riding a motor scooter is also a recreational activity, and the danger for pedestrians is roughly the same as in the case of a bicycle. His opponent argues that the case of the motor scooter more closely resembles the case of an automobile than that of a bicycle. Automobiles and motor scooters are noisy and they pollute. This comparison counts as the decisive point.

It is easy to reconstruct this argumentation by means of the analogy scheme. He who is riding a motor scooter uses A₁. He adduces a case c₁, the bicycle case, with the argument that his case c₁, the motor scooter case, is similar to the bicycle case because it shares with the bicycle case (c₂) the features of recreational activity (F₁) and a low level of danger for pedestrians (F₂). The principle of freedom of action (P₁) requires that in all cases in which these two features are present, entrance to the park ought to be permitted (Q), that is, the rule

\[(R₄) F₁ \land F₂ \rightarrow Q\]

ought to be applied.

The reply of our driver’s opponent follows A₂ in part and goes beyond it in part. This indicates that A₂ stands in need of reformulation. The opponent follows A₂ in so far as he confines himself to saying that there exists a feature in the motor scooter case (c₁) that does not exist in the bicycle case (c₂), namely, the use of a motor, which is noisy and pollutes (F₃). ‘[P]eace and quiet in the park’ as a condition of the health and well being of ordinary visitors requires that all noisy and polluting vehicles be excluded from the park, that is, it is required that the rule

\[(R₅) F₁ \rightarrow \neg Q\]

17 Ibid., 128.
18 Ibid., 129.
be applied. This argument emphasizes a feature \( (F_i^1) \) that is characteristic of the motor scooter case \( (c_i) \), but is not present in the bicycle case \( (c_j) \). Similarity is thereby contested by pointing to a difference without thereby referring to a third case. This might be termed the negative version of \( A_2 \). In our case, however, a positive version is also at hand. The opponent can argue that the motor scooter case \( (c_i) \) is similar to the automobile case \( (c_k) \) for it shares with the former the use of a motor, which causes noise and pollution \( (F^c_k) \). If one wished to give expression to the reference to the automobile case \( (c_k) \), then instead of ‘\( F_i^1 \)’ the expression ‘\( F_k^1 \)’ might be used. ‘\( F_i^1 \)’ and ‘\( F_k^1 \)’ designate the same thing, namely, the use of a motor that causes noise and pollution, that is, \( F_i^1 = F_k^1 \). In this way, the rule of the automobile case \( (c_k) \) can be expressed by means of

\[
(R_k) \quad F_k^1 \rightarrow \neg Q,
\]

which, owing to the identity of \( F_i^1 \) and \( F_k^1 \), is equivalent to \( (R_5) \). The positive version of \( A_2 \) can now be expressed by means of the requirement that \( (R_k) \) ought also to be applied in the motor scooter case.

The fact that the counter-argument can acquire both a negative and a positive form shows that \( A_2 \) has to be reformulated. But before doing this, the objections of Brożek and Bäcker might be taken up in the light of our reconstruction of the scooter case.

### IV. Case and Rule

Both Brożek and Bäcker criticize the way in which the analogy scheme incorporates rules, in our example, the rules \( F_i^1 \land F_j^1 \rightarrow Q \) and \( F_i^1 \rightarrow \neg Q \) or \( F_k^1 \rightarrow \neg Q \). Bäcker maintains that ‘the argument in \( A_1 \) is therefore not the case but the rule’.\(^9\) Brożek’s objection is more complex. He says that if \( F_i^1 \rightarrow \neg Q \) or \( F_k^1 \rightarrow \neg Q \) is a ‘valid legal rule’, then ‘there is no need for analogical reasoning’.\(^{21}\) This, however, is not his decisive point. His decisive point is that ‘there is no legal rule saying that noisy and polluting vehicles may not enter the park. The features that both cases “share” do not constitute an antecedent of a valid legal rule.’\(^{22}\)

In the Kaufmann article, I included in \( A_1 \) the premise ‘because the rule \( F_i^1 \), …, \( F_i^w \rightarrow Q \) is valid’ (‘weil die Regel \( M_g \), …, \( M_h \rightarrow R_m \) gelte’),\(^{23}\) and \( A_2 \) contains its counterpart. Indeed, this might be interpreted, as Brożek does, as a reference to a valid legal rule. This is not, however, precisely what I mean here. It is possible that an interpreter of the precedent case \( c_j \) is the very first one to identify the features shared, or not shared, that are adduced in \( A_1 \) or \( A_2 \). Thus,

---

20 Brożek uses another notation oriented on the German version in the Kaufmann article.
21 Brożek, ‘Analogy in Legal Discourse’ (n. 4), 199.
22 Ibid.
(R₃) Bicycles may enter the park

may be a valid legal rule established by precedent, whereas

(R₄′) Vehicles that serve a recreational purpose \((F_i')\) and represent no great danger to pedestrians \((F_j')\) may enter the park \((Q)\)

may be a rule that has, up until now, not been established by any social fact. In this respect, Brożek is right in maintaining that the ‘features that both cases “share” do not constitute an antecedent of a valid legal rule’. But this does not mean that the shared features adduced in A₁ do not stand in a necessary relation to a legally relevant rule. To adduce features as reasons for a certain legal consequence means to presuppose a rule containing them as antecedents. This is a corollary of the principle of universalizabilty.

It has often been remarked that cases have an unlimited number of features, and this suggests that they very often share one feature or another, as it were, randomly. Such a random sharing, however, has no legal relevance. Traffic signs requiring one to stop and motor scooters may both be made of metal, but this does not imply that one has to stop in front of a motor scooter. The features in A₁ and A₂ acquire their relevance by virtue of the fact that they are related to reasons undergirding the rules that contain them as antecedents. These reasons normally have the character of principles. In this way, cases, rules, and principles are intrinsically connected. Comparison or analogy is an argument structure that unites these three dimensions. In order to avoid misunderstanding, the validity-clauses in A₁ and A₂ ought to be changed, namely by substituting the formulations ‘because the rule \(F_1', \ldots, F_n' \rightarrow Q\) is valid’ and ‘because the rule \(F_1', \ldots, F_n' \rightarrow \neg Q\) is valid’ by the clauses ‘because there are reasons for the rule \(F_1', \ldots, F_n' \rightarrow Q\)’ and ‘because there are reasons for the rule \(F_1', \ldots, F_n' \rightarrow \neg Q\). This may help to make clear that comparison and, with it, analogy is a matter of argument – or, to be more precise, a matter of competing arguments.

On this basis, an answer can be given to Bäcker’s objection that it is only the rule in A₁, not the case, that counts. The case behind the rule is important for two reasons. The case may support the rule, but it also has the wherewithal to jeopardize the rule. After the decision of the court to the effect that bicycles may enter the park, the legal situation has changed. One who rides a motor scooter, having never before believed that he would be allowed to enter the park, acquires a new sense of hope. The mere idea that bicycles might be allowed in the park would not have given rise to this. This is the real or factual dimension of precedents. The case represents, however, also, a danger for the rule. A main point of the bicycle case is that bicycles represent no great danger to pedestrians. Severe accidents, in which the fact that an approaching bicycle makes no noise has played a decisive role, may, of course, undermine the assumption that bicycles are of little danger to pedestrian. The old case rule

---

24 Brożek, ‘Analogy in Legal Discourse’ (n. 4), 199.
26 Ibid., 92.
(R_3) Bicycles may enter the park
can no longer be upheld in the new case, which now has to be decided, and the rule

(R_4') Vehicles that serve a recreational purpose \( (F'_1) \) and represent no great danger to pedestrians \( (F'_2) \) may enter the park \( (Q) \),

far from being a rule that can be related to a concrete case, is now only an abstract idea. This dialectic of supporting and jeopardizing shows the essential case-dependence of rules that are used in comparisons. For this reason, actually decided cases, that is, precedents, are not only heuristic tools (as invented cases, that is, cases introduced merely as examples, may be). They have, due to the authoritative character of precedent, a genuine status in legal argumentation.

V. Case and Principle

As already noted, Brożek raises a further objection to the analogy scheme as expressed by A_1 and A_2. He takes the line that this scheme says nothing about how to resolve the conflict stemming from the application of both A_1 and A_2, and nothing either on how comparison or analogy might be related to both subsumption and balancing.

Indeed, the scheme as such says nothing about the questions raised by Brożek, but this is not the task of the scheme qua scheme. With respect to subsumption, something has already been said above where the relationship between cases and rules is considered. Here only the relationship between the analogy scheme and principles, that is, between comparison and balancing, is of interest.

By appeal to weighing Brożek proposes to answer the question as to which of the competing similarities is decisive, and I fully agree with him here. The analogy scheme can work only if it is connected with balancing as the basic form of the application of principles. The question is how to understand this.

It has already been mentioned that the features in A_1 and A_2 acquire their relevance by virtue of the fact that they are related to reasons undergirding the rules that contain them as antecedents, and that these reasons normally have the character of principles. This means that the subjects of balancing are the principles that support the selection of certain features \( F'_1, ..., F'_r \) or \( F'_i, ..., F'_l \) as reasons for the legal consequences \( Q \) or \( \neg Q \) and, in this way, as antecedents of the rules \( F'_1, ..., F'_r \rightarrow Q \) and \( F'_i, ..., F'_l \rightarrow \neg Q \). In our example these rules have acquired the following forms:

\[
(R_4) \ F'_1 \wedge F'_2 \rightarrow Q \text{ (permission)}
\]

27 Brożek, ‘Analogy in Legal Discourse’ (n. 4), 199.
and

\((R_3) \ F^i_1 \rightarrow \neg \ Q \) (prohibition).

\((R_4)\) is supported by the principle of freedom of action \((P_1)\), which includes recreational activities, and \((R_5)\) is supported by the principle of peace and quiet in the park \((P_2)\). \(P_1\), taken alone, requires the legal consequence of \((R_3)\), that is, \(Q\); \(P_2\) taken, again, alone, the legal consequence of \((R_5)\), that is, \(\neg Q\). The fact that \(Q\) and \(\neg Q\) stand in contradiction to one another shows that the principles \(P_1\) and \(P_2\) collide in the motor scooter case.

A collision of principle has to be resolved – if none of the competing principles is to be abandoned once and for all – by establishing a concrete relation of precedence. A relation of precedence is concrete if principle \(P_i\) takes precedence over principle \(P_j\) not absolutely or unconditionally but only under certain circumstances or conditions \((C)\).

I will assume that in the motor scooter case the principle of peace and quiet in the park \((P_2)\) takes precedence over the principle of freedom of action \((P_1)\). This concrete relation of precedence has to be justified along the lines described by the weight formula, but this matter will not be considered any further here. The only point of interest here is how the analogy scheme is connected with balancing, and this turns on the question of what in the analogy scheme is to be substituted for \(C\) in the concrete relation of preference:

\[
(1) \ (P_2 \ P_1) \ C.
\]

Three possibilities present themselves. The first is simply to identify \(C\) with a short description of the case \(c_i\), that is, with some such line as ‘motor scooter in a park’. If this short description of the case is represented by \(C_i\), the relation of preference acquires the following form:

\[
(2) \ (P_2 \ P_1) \ C_i.
\]

(2) might be termed the ‘case rule’. The other two constructions proceed from the case to its features. The first one simply refers to the preceding – and therefore decisive – feature or features, in our case, to \(F^i_1\):

\[
(3) \ (P_2 \ P_1) \ F^i_1.
\]

This is the ‘preceding feature rule’. The most complex construction substitutes the conjunction of all relevant features, in our case, the conjunction of \(F^i_1, F^i_2, \text{ and } F^i_3\), for \(C\):

\[
(4) \ (P_2 \ P_1) \ F^i_1 \land F^i_2 \land F^i_3.
\]

\[29\] Alexy, ‘The Weight Formula’ (n. 13).
Two or Three?

This might be termed the ‘relevant feature rule’.

In many cases a reference to mere case rules or preceding feature rules may suffice. In hard cases, however, the relevant feature construction will be indispensable. For it is this construction alone that completely reveals what has been accomplished by connecting comparison and balancing. In this sense, it is only the relevant feature rule that reveals the deep structure of the case.

VI. Reformulation of the Analogy Scheme

One reformulation of the analogy scheme has already been introduced: the substitution of the clauses ‘because the rule $F^i_1, \ldots, F^i_n \rightarrow Q$ is valid’ in $A_1$ and ‘because the rule $F^j_1, \ldots, F^j_n \rightarrow \neg Q$ is valid’ in $A_2$ by ‘because there are reasons for the rule $F^i_1, \ldots, F^i_n \rightarrow Q$ respectively ‘because there are reasons for the rule $F^j_1, \ldots, F^j_n \rightarrow \neg Q$.

A second reformulation has only been adumbrated: the supplementation of the negative version offered in the Kaufmann article by a positive version. By way of these two reformulations, the analogy scheme acquires the following form:

$A_1$: In every case $c_i$, each case $c_j$ may be adduced with the argument that $c_i$ shares with $c_j$ the features $F^i_1, \ldots, F^i_n$, and that $c_i$, for that reason and because there are reasons for the rule $F^i_1, \ldots, F^i_n \rightarrow Q$, ought to be treated, as $c_j$, to the effect that $Q$.

$A_2$: In each case in which an argument of the form $A_1$ is put forward, two counter-claims may be raised:

$A_{2.1}$: It may be claimed that $c_i$ is distinguished by the features $F^i_1, \ldots, F^i_n$ from $c_j$, and that $c_i$, for that reason and because there are reasons for the rule $F^j_1, \ldots, F^j_n \rightarrow \neg Q$, ought to be treated, in contradistinction to $c_j$, to the effect that $\neg Q$.

$A_{2.2}$: It may be claimed that $c_i$ shares with $c_k$ the features $F^k_1, \ldots, F^k_n$, and that $c_i$, for that reason and because there are reasons for the rule $F^k_1, \ldots, F^k_n \rightarrow \neg Q$, ought to be treated, as $c_k$, to the effect that $\neg Q$.

VII. The Basic Character of the Analogy Scheme

In section II of the paper, I remarked that a scheme has to have three distinctive characteristics in order to qualify as a basic scheme: It must be formal, necessary, and specific. Two criteria identify the analogy scheme as formal. The first is that the scheme says nothing about which features $F^i_1, \ldots, F^i_n$, $F^j_1, \ldots, F^j_n$, and $F^k_1, \ldots, F^k_n$ may figure as protases of the rules to which $A_1$ and $A_2$ refer – and, in this connection, says nothing about which features are to be classified as relevant. The second criteria that concerns the formal character is this. The scheme says nothing on the question of whether the argument according to $A_1$ or the argument according to $A_2$ prevails – that is, it says nothing on the question of what features are decisive. The formal
character is confirmed by the fact that comparison may well begin with $A_{2,2}$. $A_{2,2}$, then, switches roles with $A_1$. $A_{2,1}$ would refer to distinguishing features with respect to $c_k$, that is, $c_k$ would have to play the role of $c_j$ in $A_1$. The *apodosis* of the rule in $A_{2,1}$, then, would be $Q$ and not $\neg Q$. The necessity of the analogy scheme stems from the fact that it is not possible to refer in a rational way to other cases without using the scheme. Its specific character, finally, stems from the dialectic of reference to features of other cases. This dialectic of reference to features of other cases finds its expression in the diametrical opposition of $A_1$ and $A_2$, the latter represented in the reformulated version of the analogy scheme by $A_{2,1}$ and $A_{2,2}$. Establishing positive relations between cases on the ground of shared feature and negative relations between cases on the ground of distinct features is to decide cases by determining their place in a field of cases. In this respect, the analogy scheme is a requirement of the idea of coherence. To be sure, the analogy scheme cannot achieve coherence exclusively by its own means. The dialectic of reference to features of other cases, as shown above, cannot be rationally resolved without balancing. In this sense, comparison is necessarily connected with balancing. Necessary connections between basic schemes, however, do not, as noted above, deprive them of their specific character. Where it otherwise, not even balancing itself would be specific, for it is, at the beginning as well as at the end, necessarily connected with subsumption. The analogy scheme, therefore, is not only formal and necessary, but also specific. This more than suffices to qualify comparison as a third basic operation in law.